PHYSICAL REVIEW E **79**, 057401 (2009)

Positive and negative streamers in air: Velocity-diameter relation

G. V. Naidis
Joint Institute for High Temperatures, Russian Academy of Sciences, Moscow 125412, Russia
(Received 27 January 2009; published 13 May 2009)

The paper discusses the relation between streamer velocity and diameter that follows from an analytical approach to description of the streamer head structure. It is shown that using measured data for streamer velocity and diameter one can evaluate the electric field in the streamer head. The analytical approach predicts that for positive streamers a minimum diameter exists, inversely proportional to the gas density.

DOI: 10.1103/PhysRevE.79.057401 PACS number(s): 52.80.Hc

Electrical breakdown of gases often occurs via propagation of ionization waves in a form of plasma filaments—streamers. Study of streamer dynamics is important both for understanding details of the breakdown phenomenon and for elaboration of various applications of nonthermal plasmas produced by streamer discharges. Recently a number of papers have been published on measurements of streamer parameters and simulation of streamer propagation (see [1] and references therein). New experimental information has been obtained concerning the dependence of streamer velocity and radius on the gap geometry, applied voltage, gas pressure, etc. In [2] the values of minimum diameter of positive streamers have been measured.

Many features of streamer dynamics can be understood on the basis of analytical streamer theory [3–5]. In this paper an analytical approach is used for an analysis of measured and computed parameters of streamers in air. It is shown that with the use of simultaneously measured streamer velocity and radius it is possible to evaluate the electric field in the streamer head (the parameter that governs the efficiency of production of chemically active species by streamer discharges [6]). The analytical approach also allows one to estimate the minimum diameter of positive streamers.

The typical structure of a streamer head is shown in Fig. 1 where the axial distributions of the electron number density n_e and electric field E along the direction z of streamer propagation are presented (these results correspond to conditions of Fig. 2 of [7]: for positive streamer in atmospheric-pressure air, moving in uniform electric field 5 kV/cm). The rate S_{ex} of excitation of radiating state $N_2(C^3\Pi_u)$, evaluated as $S_{ex}=K_{ex}n_{N_2}n_e$, where K_{ex} is the rate constant of excitation of $N_2(C^3\Pi_u)$ by electron impact and n_{N_2} is the number density of nitrogen molecules, is also given. The dependence of E on z in front of the streamer head, at $z>z_h$, can be rather accurately approximated by expression

$$E(z)=E_h l_f/(z-z_h+l_f), \quad (1)$$

with approximation (1) being shown in Fig. 1 by dashed-dotted line denoted as E_{appr}. [Such form of $E(z)$ was obtained in various simulations, e.g., [8].] Results of simulations show that the axial width l_f of the region of strong electric field is typically 2–3 times smaller than the streamer radius [7]. It allows one to evaluate the distribution of n_e along z in front of the head in the framework of one-dimensional (1D) approximation.

Commonly accepted streamer models describe transport of charged species on the basis of fluid approximation; the values of kinetic and transport coefficients being taken as functions of the local electric field. The validity of the local-field approximation for description of streamers was studied in a number of works (see [9,10] and references therein). It has been shown that for streamers propagating in not too strong electric fields the account of nonlocality does not lead to substantial change of calculated streamer parameters in comparison to results obtained on the basis of fluid models.

The transport equation for electrons in the fluid approximation has a form

$$\partial n_e/\partial t+\nabla(n_e\mathbf{V}_e)=(\nu_i-\nu_a)n_e+S_e,$$

$$\mathbf{V}_e=\mu_e\mathbf{E}-D_e\nabla\ \ln(n_e), \quad (2)$$

where μ_e and D_e are the mobility and diffusion coefficients, ν_i and ν_a are the ionization and attachment rates, and the nonlocal term S_e describes the generation of precursor electrons ahead of the streamer front due to photoionization. The kinetic and transport coefficients are taken as functions of E/δ, where δ is the ratio of gas number density to its normal value, corresponding to room temperature and atmospheric

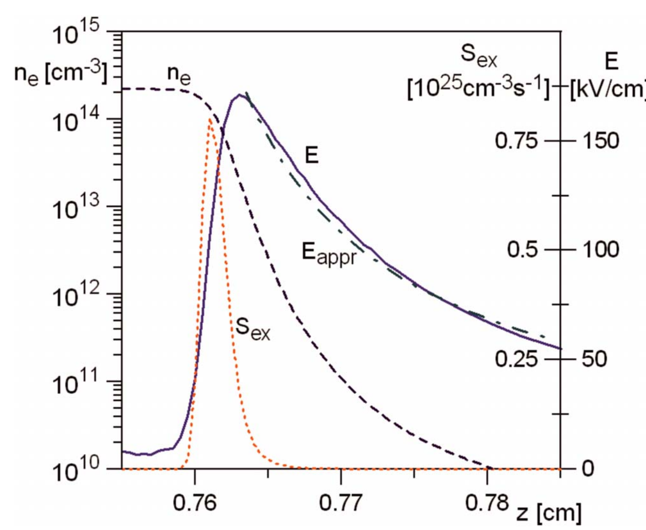

FIG. 1. (Color online) Axial distributions of electric field E, number density of electrons n_e, and rate S_{ex} of excitation of radiating state $N_2(C^3\Pi_u)$ in the streamer head region. E_{appr} is approximation (1) for the electric field distribution.

pressure. The term S_e is relatively small in the most part of ionization region ahead of the streamer and becomes effective at the external boundary of this region, where E/δ decreases to the critical value, about 30 kV/cm for air, corresponding to the equality of ν_i and ν_a. [Note that we consider streamer propagation in external electric fields much smaller than the critical one, so that the effect of external field on the distribution $E(z)$ in the ionization region is negligibly small.] The contribution of diffusion term in expression (2) for the velocity of electrons in typical conditions of streamer propagation in air is much smaller than that of the drift term. [The ratio γ of the drift and diffusion terms is evaluated as $T_e/(eE_h l_D)$, where T_e is the temperature of electrons and l_D is the diffusion length. At values of the parameters typical for streamers in air $T_e=10$ eV, $E_h/\delta=10^5$ V/cm, and $l_D\delta > 10^{-3}$ cm, one obtains $\gamma < 10^{-1}$.]

Considering quasistationary streamer propagation with constant velocity V, it is convenient to rewrite Eq. (2) in the reference system moving with the streamer head (in this system the position z_h of the electric field maximum is independent of time). In the framework of 1D approximation one obtains, neglecting the photoionization and diffusion terms, the balance equation for n_e in the ionization region at $z > z_h$,

$$-V\frac{dn_e}{dz} \mp \frac{d(n_e V_{dr})}{dz} = \alpha_{ef} V_{dr} n_e. \qquad (3)$$

Here $V_{dr}=\mu_e E$ is the absolute value of the drift velocity of electrons and $\alpha_{ef}=(\nu_i-\nu_a)/V_{dr}$ is the effective ionization coefficient. The upper and lower signs in (3) correspond to positive and negative streamers, respectively. Integration of Eq. (3) gives for n_{eh} the electron number density at $z=z_h$ the expression

$$n_{eh}(V \pm V_h) = n_{ep}(V \pm V_p)\exp\left(\int_{z_h}^{z_p} \frac{\alpha_{ef} V_{dr} dz}{V \pm V_{dr}}\right), \qquad (4)$$

where n_{ep} is the value of n_e at a point z_p corresponding to the external boundary of the ionization region and V_h and V_p are the values of V_{dr} at points z_h and z_p, respectively.

Substituting, in the integral in Eq. (4), distribution (1) for $E(z)$, one obtains

$$l_f \delta \frac{E_h}{\delta} \int_{E_p/\delta}^{E_h/\delta} \frac{d(E/\delta)}{(E/\delta)^2} \frac{\alpha_{ef}}{\delta} \frac{V_{dr}}{V \pm V_{dr}} = \ln\left(\frac{n_{eh}}{n_{ep}}\right) + \ln\left(\frac{V \pm V_h}{V \pm V_p}\right), \qquad (5)$$

where E_p is the electric field at $z=z_p$. Expression (5) relates, for given E_h/δ, the streamer parameters $l_f\delta$ and V. It follows from Eq. (5) that at high velocities of streamer propagation, $V \gg V_h$, the streamer velocity is, for given E_h/δ, proportional to $l_f\delta$. It is seen also that at $V \gg V_h$ the values of V are, for given E_h/δ and $l_f\delta$, the same for positive and negative streamers. Another consequence of Eq. (5) is an existence for positive streamers of minimum value of $l_f\delta$ corresponding to zero streamer velocity.

Numerous simulations of positive streamers in air for various pressures, gap geometries, and applied voltages give E_h/δ values in the range of 100–200 kV/cm. Much larger, more than 1 order of magnitude, are the ranges of variation

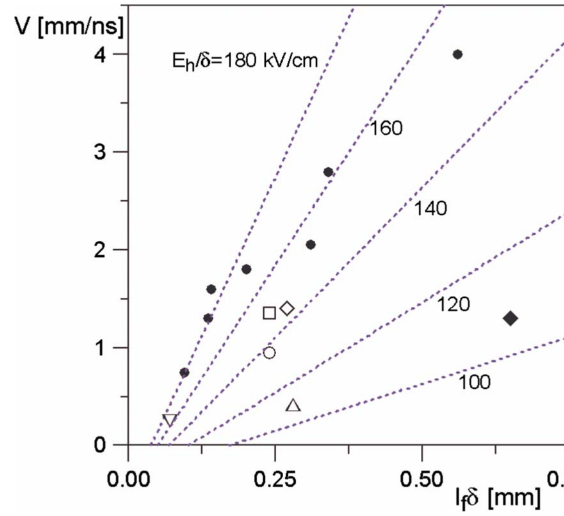

FIG. 2. (Color online) Mean positive streamer velocity V vs $l_f\delta$. Points—results of numerical simulation (in parentheses the values of E_h/δ in kV/cm are given): ● [7] (160–180), ▽ [11] (160), □ [12] (150, δ=0.4), ◇ [13] (140), ○ [14] (140), △ [15] (120), ◆ [16] (100). Lines—evaluation for various E_h/δ.

for calculated streamer velocity and radius. In Fig. 2 the values of $l_f\delta$ and V obtained in several simulations of positive streamers in air are given (the results from the works other than [12] correspond to δ=1). Obtained in these simulations values of E_h/δ, given in the parentheses in the caption to Fig. 2, vary in the range of 100–180 kV/cm. In the same figure the positive streamer velocity evaluated using Eq. (5) is given versus $l_f\delta$ for various E_h/δ. A typical value $\eta \equiv \ln(n_{eh}/n_{ep})=8$ is assumed (according to results of streamer simulations, n_{eh} is about 10^3-10^4 times larger than the electron number density n_{ep} at the external boundary z_p of ionization region, corresponding to the critical electric field). Data for α_{ef}/δ and V_{dr} in air as functions of E/δ were taken from [17]. It is seen that the simulation results agree with estimate (5) for corresponding E_h/δ. Note that the estimated streamer velocity at given $l_f\delta$ is rather sensitive to variation of E_h/δ. Hence at known V and $l_f\delta$ it is possible to estimate rather accurately the maximum electric field in streamer head.

Results of streamer simulation show that the axial width l_f of the distribution of electric field in front of the streamer head is proportional to the radial dimensions of the head region such as the electrodynamic and radiation diameters. The calculated ratio of the radiation diameter d to the width l_f varies in the range of 3–5 (e.g., [7,8,12]). In our evaluations below the mean value of this ratio, $\xi \equiv d/l_f=4$, is taken. The positive streamer velocity versus the radiation diameter for atmospheric-pressure air, estimated using expression (5) for l_f, is shown in Fig. 3. Experimental results [18,19] obtained for positive streamers in atmospheric-pressure air are also given. (For evaluation of the streamer velocity in conditions [19] we used data for the streamer head position versus time presented in the paper.) It is seen that the dependence of the velocity on the radiation diameter is not far from linear; the experimental points lying between the lines corresponding to E_h/δ=120 and 160 kV/cm.

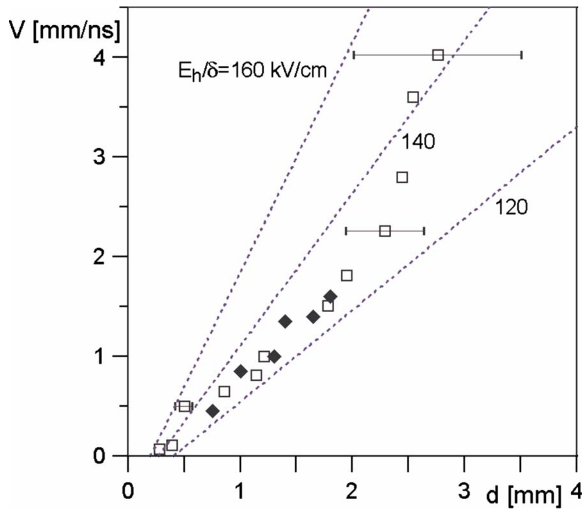

FIG. 3. (Color online) Mean positive streamer velocity V vs radiation diameter d at normal gas density. Points—experimental data □ [18], ◆ [19]. Lines—evaluation for various E_h/δ.

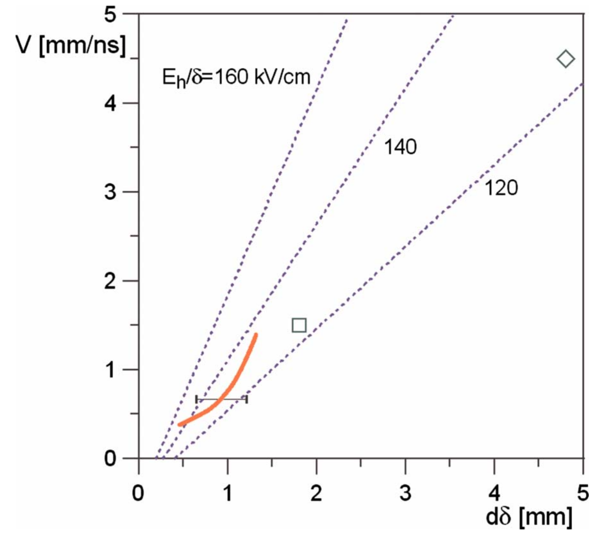

FIG. 4. (Color online) Mean positive streamer velocity V vs $d\delta$. Solid line—experimental data [12], ◇ [20], □ [22]. Dotted lines—evaluation for various E_h/δ.

In Fig. 4 the estimates of V versus $d\delta$ for positive streamers in air are compared to experimental data [12] obtained for the pressure range of 300–750 torr. The experimental data are also shown, obtained in [20] for 360 torr (streamer velocity and radius in these conditions have been evaluated in [21] by treatment of photographs presented in [20]) and those obtained in [22] for 460 torr (streamer velocity and radius are estimated using the streamer head images shown in Fig599. 7 of [22]). It is seen that results [12,20,22] correspond to the same range of E_h/δ, 120–160 kV/cm, as the data [18,19] shown in Fig. 3.

It is interesting to compare our estimates for the maximum electric field in positive streamer head with those obtained by measurement of intensities of radiation emitted by $N_2(C^3\Pi_u)$ and $N_2^+(B^2\Sigma_u^+)$ [16,21,23–25]. The latter method is based on the fact that the ratio of the radiation intensities is governed by the electric field. Note that this method, being rather accurate for spatially uniform conditions, can lead to noticeable errors at strong nonuniformity of plasma parameters such as in streamer head region. Figure 1 shows that the electric field in the maximum of excitation rate S_{ex} is about 1.5 times lower than the maximum field E_h. It follows that by measuring the electric field in streamer head via the intensity ratio integrated over the radiating plasma volume, as it was done in [16,21,23,24], one would obtain E value (corresponding to the maximums of radiation intensities) that is lower than E_h. (An attempt to account for the shift of radiation intensity maximums has been made in [25] where some axial distributions of streamer parameters in the head region were assumed.) Note also that, as it has been shown in [26], the relation between the intensity ratio and E/δ, used in most of previous works, contained a systematic error, therefore previous results should be corrected. Re-estimation, with the use of the data [26], of E/δ obtained via the intensity ratio in works [16,24] gives, for conditions of both of these works, the values 70–90 kV/cm. Close E/δ values, 82 ± 19 kV/cm, have been obtained with the same method in [21]. Assuming, on the basis of the distributions shown in Fig. 1, that E_h/δ is 1.5 times larger than E/δ estimated via the intensity ratio, one obtains E_h/δ in the range 100–140 kV/cm. This estimate agrees with the values $E_h/\delta=110-125$ kV/cm obtained in [25] with an approximate account of the mentioned shift of the distribution maximums. The above estimates for the electric field in positive streamer head, $E_h/\delta=120-160$ kV/cm, based on the diameter-velocity relation, are slightly higher than those obtained via the measured intensity ratio, 100–140 kV/cm, though the difference between the two estimates does not exceed the uncertainty of both these methods. It should be mentioned that simultaneous evaluation of the radiation diameter and velocity during streamer propagation, e.g., by treatment of photographs, is much simpler than the spectral measurements required for evaluation of the radiation intensities.

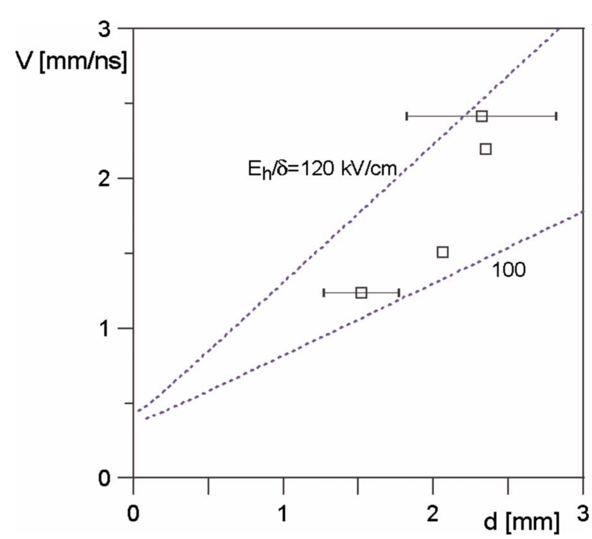

FIG. 5. (Color online) Mean negative streamer velocity V vs radiation diameter d at normal gas density. Points—experimental data [18]; lines—evaluation for various E_h/δ.

Estimates of the negative streamer velocity versus the radiation diameter in atmospheric-pressure air, obtained using Eq. (5) and the values of η and ξ presented above, are shown in Fig. 5. Experimental data [18] for negative streamers in atmospheric-pressure air are also given. They correspond to the estimates for the range of E_h/δ from 100 to 120 kV/cm. Note that the values of E_h/δ for negative streamers are smaller than those for positive streamers, in agreement with results of simulations [14,27].

The minimum radiation diameter of positive streamer $d_{\min}=\xi l_{f\,\min}$ is evaluated, according to Eq. (5) at $V=0$, as

$$d_{\min}\delta=\xi\left[\eta+\ln\left(\frac{V_h}{V_p}\right)\right]\left(\frac{E_h}{\delta}\int_{E_p/\delta}^{E_h/\delta}\frac{d(E/\delta)}{(E/\delta)^2}\frac{\alpha_{ef}}{\delta}\right)^{-1}. \quad (6)$$

Estimates (6) of $d_{\min}\delta$ for $E_h/\delta=140$ and 160 kV/cm are 0.27 and 0.20 mm, respectively. They are rather close to the measured value $d_{\min}\delta=0.20\pm0.02$ mm [2]. Equation (6) shows that $d_{\min}\delta$ decreases with growth of E_h/δ. It is known from simulations of streamer dynamics in various gases that the values of E_h/δ are typically higher in gases with lower efficiency of production of photoelectrons (e.g., [6,28,29]), therefore the minimum streamer diameters in such gases are to be typically smaller. Indeed, the measured minimum diameter of positive streamers in nitrogen [2] is about twice smaller than that in air.

In conclusion, the above consideration gives a relation between the streamer velocity V and radiation diameter d that allows one to evaluate, at known V and d, the maximum electric field in the streamer head. It follows from this relation that for positive streamers a minimum diameter $d_{\min}$ exists, inversely proportional to the relative gas number density δ, the estimate of $d_{\min}\delta$ for air being consistent with the measured value.

The author is grateful to Dr. S. Pancheshnyi for helpful discussions.

[1] U. Ebert and D. D. Sentman, J. Phys. D **41**, 230301 (2008).
[2] T. M. P. Briels, E. M. van Veldhuizen, and U. Ebert, J. Phys. D **41**, 234008 (2008).
[3] M. I. Dyakonov and V. Yu. Kachorovsky, Sov. Phys. JETP **67**, 1049 (1988).
[4] V. A. Shveigert, High Temp. **28**, 792 (1990).
[5] N. Yu. Babaeva and G. V. Naidis, in *Electrical Discharges for Environmental Purposes: Fundamentals and Applications*, edited by E. M. van Veldhuizen (Nova Science, New York, 2000), p. 21.
[6] N. Yu. Babaeva and G. V. Naidis, IEEE Trans. Plasma Sci. **26**, 41 (1998).
[7] N. Yu. Babaeva and G. V. Naidis, J. Phys. D **29**, 2423 (1996).
[8] A. A. Kulikovsky, Phys. Rev. E **57**, 7066 (1998).
[9] G. V. Naidis, Tech. Phys. Lett. **23**, 493 (1997).
[10] C. Li, W. J. M. Brok, U. Ebert, and J. J. A. M. van der Mullen, J. Appl. Phys. **101**, 123305 (2007).
[11] R. Morrow and T. R. Blackburn, J. Phys. D **35**, 3199 (2002).
[12] S. Pancheshnyi, M. Nudnova, and A. Starikovskii, Phys. Rev. E **71**, 016407 (2005).
[13] S. Pancheshnyi and A. Starikovskii, J. Phys. D **36**, 2683 (2003).
[14] A. Luque, V. Ratushnaya, and U. Ebert, J. Phys. D **41**, 234005 (2008).
[15] N. Liu, S. Celestin, A. Bourdon, V. P. Pasko, P. Segur, and E. Marode, Appl. Phys. Lett. **91**, 211501 (2007).
[16] Y. Kim, W. S. Kang, J. M. Park, S. H. Hong, Y.-H. Song, and S. J. Kim, IEEE Trans. Plasma Sci. **32**, 18 (2004).
[17] J. W. Gallagher, E. C. Beaty, J. Dutton, and L. C. Pitchford, J. Phys. Chem. Ref. Data **12**, 109 (1983).
[18] T. M. P. Briels, J. Kos, G. J. J. Winands, E. M. van Veldhuizen, and U. Ebert, J. Phys. D **41**, 234004 (2008).
[19] G. J. J. Winands, Z. Liu, A. J. M. Pemen, E. J. M. van Heesch, and K. Yan, J. Phys. D **41**, 234001 (2008).
[20] P. P. M. Blom, Ph.D. thesis, Technical University Eindhoven, 1997.
[21] T. M. P. Briels, Ph.D. thesis, Technical University Eindhoven, 2007.
[22] M. M. Nudnova and A. Yu. Starikovskii, J. Phys. D **41**, 234003 (2008).
[23] N. Spyrou and C. Manassis, J. Phys. D **22**, 120 (1989).
[24] S. V. Pancheshnyi, S. V. Sobakin, S. M. Starikovskaya, and A. Yu. Starikovskii, Plasma Phys. Rep. **26**, 1054 (2000).
[25] Yu. V. Shcherbakov and R. S. Sigmond, J. Phys. D **40**, 474 (2007).
[26] P. Paris, M. Aints, F. Valk, T. Plank, A. Haljaste, K. V. Kozlov, and H.-E. Wagner, J. Phys. D **38**, 3894 (2005).
[27] N. Yu. Babaeva and G. V. Naidis, IEEE Trans. Plasma Sci. **25**, 375 (1997).
[28] S. V. Pancheshnyi, S. M. Starikovskaya, and A. Yu. Starikovskii, J. Phys. D **34**, 105 (2001).
[29] G. V. Naidis, J. Phys. D **40**, 4525 (2007).